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# MASS TRANSFER EFFECT ON MHD FREE CONVECTIVE FLOW ON VISCO-ELASTIC FLUID THROUGH POROUS MEDIUM OVER AN OSCILLATING POROUS PLATE WITH HEAT AND MASS FLUX

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## ABSTRACT

Effect of Mass transfer on MHD free convective flow on visco-elastic fluid through porous medium over an oscillating porous plate with heat and mass flux has been studied. The dimensionless governing equations are solved using oscillatory flow equation having both harmonic and non harmonic parts. The velocity near the plate with slip flow regime is assumed to vary with respect to time. The influences of the various parameters on the flow field, mass concentration field, Temperature field and skin friction are extensively discussed from graphs and table.

**KEYWORDS:** Convective, Oscillating, Porous Medium, Visco-Elastic, MHD Mass Transfer, Second Grade Fluid, Heat & Mass Flux

## 1. INTRODUCTION

The study related to free convective flow in presence of heat source has drawn clean attention of many researchers in last few decades, because of its wide application in astrophysics and comical studies. These types of flows play a vital role in chemical engineering, aerospace technology etc. under unsteady oscillating flows have application in many other technological fields.

The analysis of visco-elastic flow is one of the important fields of fluid dynamics. The complex stress –strain relationships of visco-elastic fluid mechanisms are used in geophysics, chemical engineering, hydrology, soil physics and pulp technology. The mechanisms of elastic-viscous boundary layer flow are used in various manufacturing process like fabrication of adhesive tapes, coating layers into girid surface etc.

Analytical studies of forced, free and mixed convection flow of viscous incompressible fluid along a vertical surface in the presence of magnetic field have been conducted by Merkinn [1] and Sparrow [2]. The influence on an electrically conducting viscous incompressible fluid has vastly application in the manufacture of Rayon and Nylon, purification of crude oil and textile industry, because of its application for MHD natural convection flow in the nuclear engineering where convection aids the cooling of reactors, the natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of strong magnetic field has been studied by several authors such as Crammer [4], Hossain [5] and Kuiken [3]. Singh and Singh [7] discussed the MHD free convection flow and mass transfer past a flat plate. Al-Qadat and Al-Azab [8] studied the influence of chemical reaction on transient MHD free convective flow over a moving vertical plate. Palani and Srikanth [9] explained the mass transfer effects on MHD flow past a semi infinite vertical plate. Two –dimensional MHD oscillatory flow along a uniformly moving infinite vertical porous plate bounded by porous medium is studied by Ahmed et. Al [6]. Senapati et.al [10] have analysis the magnetic effect on mass and heat transfer of a hydrodynamic flow past a vertical oscillating plate in presence of chemical reaction.

This paper deals with the study of mass transfer effect on MHD free convective flow on visco-elastic fluid through porous medium over an oscillating porous plate with heat and mass flux.

## 2. FORMULATION OF PROBLEM

Let us consider the electrically conducting incompressible second grade free convective fluid through porous medium which occupy semi infinite region over an oscillating porous with constant heat, mass flux and a constant suction $v_0$. X-axis is taken along the plate in the moving direction of the plate and a magnetic field of uniform strength $B_0$ is applied perpendicular to the plate along Y-axis. $T'_\infty$ is the temperature and $C'_\infty$ is the mass concentration of fluid. Since the plate is infinite dimensional along x and z direction, so all the physical quantities are the function of y and t only. By applying Boussinesq approximation and taking viscous dissipation in porous region in account, the governing equations are given below:

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} - v'_0 \frac{\partial u'}{\partial y'} = g\beta(T - T_\infty) + g\beta_c(C - C_\infty) + \left(\nu + \alpha' \frac{\partial}{\partial t'}\right)\frac{\partial^2 u'}{\partial y'^2} - \frac{\nu}{k_0}u' - \frac{\sigma {B_0}^2}{\rho}u' \tag{2}$$

$$\frac{\partial T'}{\partial t'} - v'_0 \frac{\partial T'}{\partial y'} = k\left(\nu + \alpha' \frac{\partial}{\partial t'}\right)\frac{\partial^2 T}{\partial y''^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'} - v'_0 \frac{\partial C'}{\partial y'} = D\left(\nu + \alpha' \frac{\partial}{\partial t'}\right)\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

With the following boundary conditions

$$\left.\begin{aligned} u' = U_{0,}e^{i\omega' t'}, \frac{\partial T'}{\partial y'} = -\frac{q}{k}, \frac{\partial C'}{\partial y'} = -\frac{m}{D} \; at\; y' = 0 \\ u' \to 0, T' \to T'_\infty, C' \to C'_\infty \; as\; y' \to \infty \end{aligned}\right\} \tag{5}$$

Where g is acceleration due to gravity, $T'$ is the temperature of the fluid, $C'$ is the mass concentration of the fluid, $\beta$ is the coefficient of thermal expansion, $\beta_c$ is the volumetric expansion coefficient, $\nu$ is the kinematic viscosity of the fluid, $k$ thermal is the their conductivity, ρ is the density of the fluid, $k_0$ is the permeability of porous medium, $C_p$ is the specific heat at constant pressure, D is the mass diffusion coefficient, q is the heat flux per unit area and m is the mass flux per unit area.

Let us introduce the following non-dimensional quantities

$$\left.\begin{aligned} y = \frac{U_0 y'}{\nu}, u = \frac{u'}{U_0}, t = \frac{U_0^2 t'}{\nu}, \omega = \frac{\nu\omega'}{U_0^2}, \theta = \frac{(T' - T'_\infty)U_0 k}{q\nu}, C = \frac{(C' - C'_\infty)U_0 D}{m\nu}, \\ Pr = \frac{\nu}{k}, Sc = \frac{\nu}{D}, K = \frac{U_0^2 k_0}{\nu^2}, M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, Gr = \frac{g\beta q \nu^2}{U_0^4 k}, Gm = \frac{g\beta_c m \nu^2}{U_0^4 D}, \alpha = \alpha' \frac{U_0^2}{\nu^2}, v_0 = \frac{v'_0}{U_0} \end{aligned}\right\} \tag{6}$$

Where $Gr$ is Grashof number, $Gm$ modified Grashof number, M is magnetic number, Pr is the Prandtl number, Sc is Schmidt number K permeability parameter porous medium, $\propto$ is normal stress parameter

Then using equation (6), equations (2) to (4) with boundary condition (5), reduce to

$$\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y} = Gr\theta + GmC + \left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y'^2} - \left(\frac{1}{K}+M\right)u \tag{7}$$

$$\frac{\partial \theta}{\partial t} - v_0\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 \theta}{\partial y'^2} \tag{8}$$

$$\frac{\partial C}{\partial t} - v_0\frac{\partial C}{\partial y} = \frac{1}{Sc}\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial^2 C}{\partial y'^2} \tag{9}$$

With the following boundary conditions

$$u = e^{i\omega t}, \frac{\partial\theta}{\partial y} = -1, \frac{\partial C}{\partial y} = -1 \; at\; y = 0 \tag{10}$$

$$u \to 0, \theta \to 0, C \to 0 \; as\; y \to \infty$$

## 3. SOLUTION OF PROBLEM

To solve the solution of the problem for velocity, temperature and concentration field are assumed to be in the following field

$$\left.\begin{aligned} u &= u_0 + u_1 e^{i\omega t}\\ \theta &= \theta_0 + \theta_1 e^{i\omega t}\\ C &= C_0 + C_1 e^{i\omega t}\end{aligned}\right\} \tag{11}$$

Substituting the expressions for u,$\theta$ and C in equation (7) to (9) and boundary condition (10), and separating the harmonic and non harmonic parts the following equations are obtained.

$$\frac{\partial^2 u_1}{\partial y^2}(1+\propto i\omega) + v_0\frac{\partial u_1}{\partial y} = i\omega u_1 - Gr\theta_1 - GmC_1 \tag{12}$$

$$\frac{\partial^2 u_0}{\partial y^2} + v_0\frac{\partial u_0}{\partial y} = \left(M+\frac{1}{K}\right)u_0 - Gr\theta_0 - GmC_0 \tag{13}$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + Prv_0\frac{\partial \theta_0}{\partial y} = 0 \tag{14}$$

$$\frac{\partial^2 \theta_1}{\partial y^2}(1+\propto i\omega) + Prv_0\frac{\partial \theta_0}{\partial y} - \alpha i\omega\theta_1 = 0 \tag{15}$$

$$\frac{\partial^2 C_0}{\partial y^2} + Scv_0\frac{\partial C_0}{\partial y} = 0 \tag{16}$$

$$\frac{\partial^2 C_1}{\partial y^2}(1+\propto i\omega) + Scv_0\frac{\partial C_0}{\partial y} - \alpha i\omega C_1 = 0 \tag{17}$$

With boundary conditions

$$\left.\begin{aligned} &u_0 = 0, u_1 = 0, \frac{\partial\theta_0}{\partial y} = -1, \frac{\partial\theta_1}{\partial y} = 0, \frac{\partial C_0}{\partial y} = -1, \frac{\partial C_1}{\partial y} = 0 \; for\; y = 0\\ &u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, C_0 = 0, C_1 = 0 \; for\; y \to \infty\end{aligned}\right\} \tag{18}$$

By solving the equations (14) to (21) using the condition (22), we get

$$C = \frac{1}{Sc\, v_0}e^{-Scv_0 y} + (b_3 e^{-a_3 y})e^{i\omega t} \tag{19}$$

$$\theta = \frac{1}{Pr\, v_0}e^{-Prv_0 y} + (b_2 e^{-a_2 y})e^{i\omega t} \tag{20}$$

$$u = (b_1 e^{-a_1 y} - b_4 e^{-PrV_0 y} - b_5 e^{-ScV_0 y}) + (b_8 e^{-a_4 y} - b_6 e^{-a_2 y} - b_7 e^{-a_3 y})e^{i\omega t} \quad (21)$$

Let, $u_1(y,t) = M_r + iM_i$, then

$$u\,(y,t) = u_0(y,t) + u_1(y,t)e^{i\omega t}$$

$$= [u_0 + (M_r cos\omega t - M_i sin\omega t)] + i[M_r sin\omega t + M_i cos\omega t] \quad (22)$$

The real part of $u(y,t)$

$$u_r = u_0 + (M_r cos\omega t - M_i sin\omega t) \quad (23)$$

This expression takes simplest form for $\omega t = \frac{\pi}{2}$ and given by $u_r = u_0 - M_i$ (24)

Let $\theta_1(y,t) = \theta_r + i\theta_i$ and $C_1(y,t) = C_r + iC_i$, then the real part of

$$\theta(y,t) = \theta_r = \theta_0 + (\theta_r cos\omega t - \theta_i sin\omega t) \quad (25)$$

and the real part of $C(y,t) = C_r = C_0 + (C_r cos\omega t - C_i sin\omega t)$ (26)

This expression takes simplest form for $\omega t = \frac{\pi}{2}$ and given by $C_r = C_0 - C_i$ (27)

In view of equation (21), the skin friction

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y} + e^{i\omega t}\frac{\partial u_1}{\partial y}\right)_{y=0}$$

$$= (-b_1 a_1 + Prv_0 b_4 + Scv_0 b_5) + (-b_8 a_4 + a_2 b_6 + a_3 b_7)e^{i\omega t}$$

$$= (-b_1 a_1 + Prv_0 b_4 + Scv_0 b_5) + |F|e^{i(\omega t+\varphi)} \quad (28)$$

Where $|F| = \sqrt{F_r^2 + F_i^2}, tan\varphi = \frac{F_i}{F_r}, where\ F_r\ and\ F_i\ are\ real\ part\ and\ imaginary\ part\ of\ of\ F$

In view of equation (20), the Nusselt number

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$= -\left(\frac{\partial \theta_0}{\partial y} + e^{i\omega t}\frac{\partial \theta_1}{\partial y}\right)_{y=0}$$

$$= 1 + (a_2 b_2)e^{i\omega t}$$

$$= 1 + |Q|e^{i(\omega t+\psi)} \quad (29)$$

Where $|Q| = \sqrt{S_1^2 + S_2^2}, tan\psi = \frac{S_2}{S_1}$ and $S_1\ and\ S_2\ are\ real\ part\ and\ imaginary\ part\ of\ of\ Q$

In view of equations (19), the Sherwood number

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$= -\left(\frac{\partial C_0}{\partial y} + e^{i\omega t}\frac{\partial C_1}{\partial y}\right)_{y=0}$$

$$= 1 + (a_3 b_3)e^{i\omega t}$$

$$= 1 + |T|e^{i(\omega t+\alpha)} \quad (30)$$

Where $|T| = \sqrt{S_3^2 + S_4^2}$, $tan \propto = \frac{S_4}{S_3}$ *and* $S_3$ *and* $S_4$ *are real part and imaginary part of of* $T$

Where $b_1 = b_4 + b_5, a_1 = \frac{V_0+\sqrt{V_0^2+4\left(M+\frac{1}{K}\right)}}{2}, a_2 = \frac{P_rV_0+\sqrt{P_r^2V_0^2+4\alpha^2\omega^2-4P_r\omega i}}{2(1+\alpha\omega i)}$

$$a_3 = \frac{S_cV_0 + \sqrt{S_c^2V_0^2 + 4\alpha^2\omega^2 - 4S_c\omega i}}{2(1+\alpha\omega i)}, b_2 = \frac{1}{a_2}, b_3 = \frac{1}{a_3}, a_4 = \frac{V_0 + \sqrt{V_0^2 + (1+\alpha\omega i)\omega i}}{2(1+\alpha\omega i)}, b_4$$

$$= \frac{Gr}{PrV_0\left(P_r^2V_0^2 - PrV_0^2 - (M + \left(\frac{1}{K}\right))\right)} b_5 = \frac{Gm}{ScV_0\left(S_c^2V_0^2 - ScV_0^2 - (M + \left(\frac{1}{K}\right))\right)} b_6$$

$$= \frac{Grb_2}{(1+i\alpha\omega)a_2^2 - V_0a_2 - i\omega}, b_7 = \frac{Gmb_3}{(1+i\alpha\omega)a_3^2 - V_0a_3 - i\omega} b_8 = b_6 + b_7$$

## 4. RESULTS AND DISCUSSIONS

In this paper we have studied the Mass transfer effect on MHD free convective flow on visco-elastic fluid through porous medium over an oscillating porous plate with heat and mass flux. The effect of the parameters Gr, Gm, M, K, $v_0$, Sc, Pr, ω, t and $\alpha$ on flow characteristics have been studied and shown by means of graphs and tables. In order to have physical correlations, we choose suitable values of flow parameters. The graphs of velocities, heat and mass concentration are taken w. r. t. y and the value of Shearing stress is shown in the table for different values of flow parameters.

Velocity profiles: The velocity profiles are depicted in Figs 1-4. Figure-(1) shows the effect of the parameters Sc and Pr on velocity at any point of the fluid, when M=2, K=2, Gr=2, Gm=2, $\alpha = 2$, $v_0$=2, t=0.2 and $\omega = 1$. It is noticed that the velocity decreases with the increase of Schmidt number (Sc), where as fluctuate with the increase of Prandtl number (Pr).

Figure-(2) shows the effect of the parametersα, $v_0$, t and ω on velocity at any point of the fluid, when M=2, K=2, Gr=2, Gm=2, $Pr = 0.7$ and $Sc = 0.23$. It is noticed that the velocity decreases with the increase of normal stress parameter (α), suction parameter ($v_0$) and oscillatory frequency (ω ), whereas increases with the increase of time (t).

Figure-(3) shows the effect of the parameters Gr and Gm on velocity at any point of the fluid, when M=2, K=2, Pr=0.7, Sc=0.23, $\alpha = 2$, $v_0$=2, t=0.2 and $\omega = 1$. It is noticed that the velocity increases with the increase of modified Grashof number (Gm), whereas fluctuated with the increase of Grash of number (Gr).

Figure-(4) shows the effect of the parameters K and M on velocity at any point of the fluid, when Sc=0.23, Pr=0.7, Gr=2, Gm=2, $\alpha = 2$ $v_0$=2, t=0.2 and $\omega = 1$. It is noticed that the velocity increases with the increase of permeability parameter porous medium (K), whereas decreases with the increase of Magnetic parameter (M).

Mass concentration profile: The mass concentration profiles are depicted in Figs 5-6. Figure-(5) shows the effect of Sc, α and $v_0$ on Mass Concentration profile when t=0.2 and$\omega = 1$. It is noticed that the mass concentration decreases in the increases of Schmidt number (Sc) and normal stress parameter (α), whereas decreases initially and then increase with the increase suction parameter ($v_0$). In fig-6, shows the effect of Oscillation (ω) and time(t)that mass concentration decreases initially and then increase with the increase both parameters.

Temperature profile: The Temperature profiles is depicted in Figs 7-8 Figure-(7) shows the effect of Pr, α and $v_0$ on Temperature profile, when t=0.2 and $\omega = 1$. It is noticed that the Temperature fall in the increase of normal stress parameter (α), whereas fluctuate in the increase Prandtl number (Pr) and suction parameter ($v_0$). In fig-8 shows the effect of Oscillation (ω) and time(t) that, the Temperature fall initially and then raise with the increase both parameters.

Table-(1) shows the effects of different parameters on Shearing stress. It is noticed that shearing stress increases in the increase of Grashoff number (Gr), modified Grashoff number (Gm), Normal stress parameter (α), Oscillation (ω) and Schmidt number (Sc). Whereas decreases with the increase of Prandtl number (Pr) and time (t), but it is fluctuate for suction parameter ($v_0$) and no effect for permeability parameter porous medium (K) and Magnetic parameter (M).

**Table 1: Effect of Different Parameters on Shearing Stress**

| Parameter | Values of Parameters | Harmonic Part of Skin Friction | Tangent Value of Phase Angle |
|---|---|---|---|
| Sc | 0.23 | 0.3259 | -0.1950 |
| | 0.3 | 0.3321 | -0.1769 |
| | 0.6 | 0.3392 | -0.1167 |
| Pr | 1 | 0.3107 | -0.1741 |
| | 7 | 0.1736 | -0.2859 |
| Gr | 3 | 0.4098 | -0.1761 |
| | 4 | 0.4938 | -0.1636 |
| Gm | 4 | 0.4893 | -0.2273 |
| | 6 | 0.6442 | -0.2439 |
| M | 3 | 0.3259 | -0.1950 |
| | 4 | 0.3259 | -0.1953 |
| K | 4 | 0.3259 | -0.1950 |
| | 5 | 0.3259 | -0.1953 |
| $V_0$ | 3 | 0.0869 | 2.5638 |
| | 4 | 0.2558 | 1.0075 |
| $\alpha$ | 3 | 0.4519 | 0.1252 |
| | 4 | 0.4702 | 0.2247 |
| $\omega$ | 1.5 | 0.4897 | 0.2323 |
| | 2 | 0.4847 | 0.5344 |
| t | 0.8 | 0.3269 | 0.0074 |
| | 1 | 0.3256 | 0.2104 |

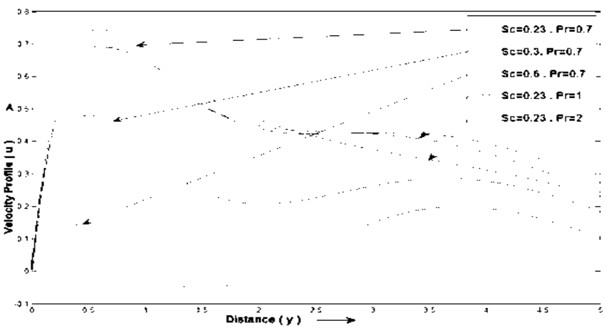


**Figure 1: Effect of SC and Pron velocity Profile when M=2, K=2, Gr=2, Gm=2, $\alpha = 2$, $v_0$=2, t=0.2 and $\omega = 1$**

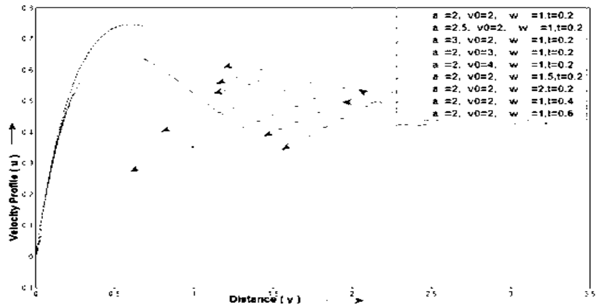


**Figure 2: Effect of $\alpha$, $v_0$, T and $\omega$ On Velocity Profile when M=2, K=2, Gr=2, Gm=2, $Pr = 0.7$ and $Sc = 0.23$.**

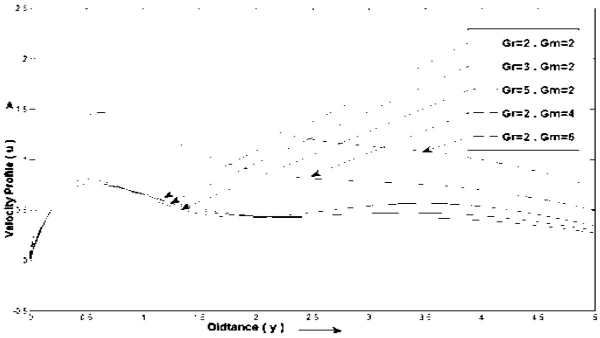


**Figure 3: Effect of Gr and Gm on Velocity Profile when M=2, K=2, Pr=0.7, Sc=0.23, $\alpha = 2$, $v_0$=2, T=0.2 And $\omega = 1$**

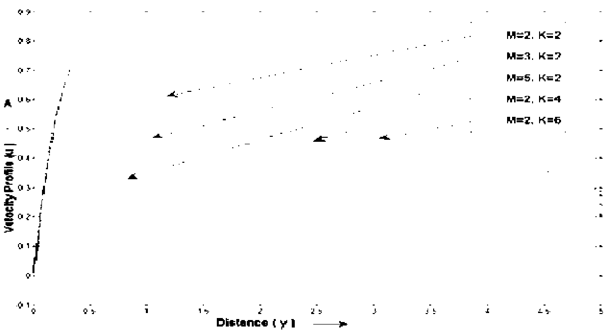


**Figure 4: Effect of K and M on Velocity Profile when Sc=0.23, Pr=0.7, Gr=2, Gm=2, $\alpha = 2$, $v_0$=2, T=0.2 and $\omega = 1$**

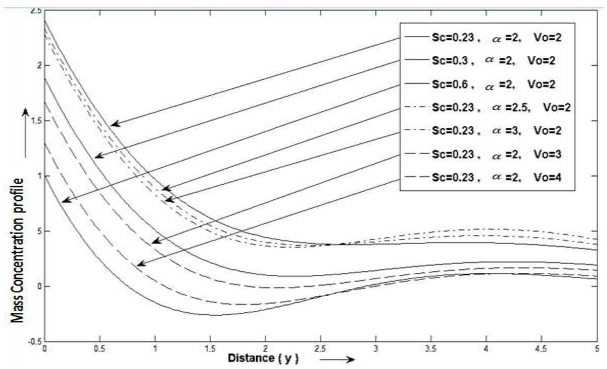


**Figure 5: Effect of Sc, $\alpha$ and $v_0$ on Mass Concentration Profile when T=0.2 and $\omega = 1$**

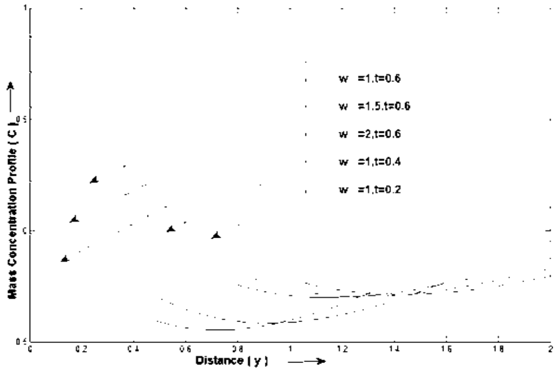


**Figure 6: Effect of $\omega$ $and$ $t$ on Mass Concentration Profile when Sc=0.23, $\alpha = 2$ and $v_0 = 2$**

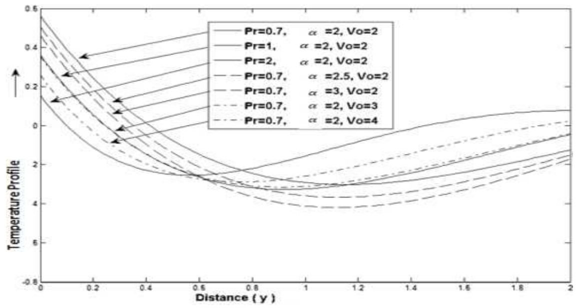


**Figure 7: Effect of Pr, $\alpha$ and $v_0$ on Temperature profile when t=0.2 and $\omega = 1$**

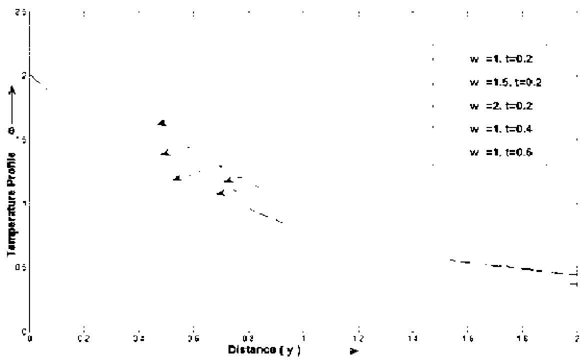


**Figure 8: Effect of $\omega$ $and$ $t$ on Temperature Profile when Pr=0.7, $\alpha = 2$ and $v_0 = 2$.**

## CONCLUSIONS

This paper deals with the study of the effect of Mass transfer effect on MHD free convective flow on visco-elastic fluid through porous medium over an oscillating porous plate with heat and mass flux. From the above observation is found that(1) The velocity increases for the increasing of K, Gm, t and Sc, also decreases for the increasing of M, $\alpha, v_0$, Gr& $\omega$.(2)Temperature fall for the increase of $\alpha$.(3) Mass concentration increases for the increase of $\alpha$ and Sc. So Magnetic force dominated by viscous force and momentum diffusivity dominate the mass diffusivity leads the increase of velocity and mass concentration in the flow field.

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