

COMPUTATION OF THE MAGNETIC CRITICAL POINT EXPONENT (β) OF FERROMAGNETS USING 1 – DIMENSIONAL ISING MODEL

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ABSTRACT

The magnetic critical point exponent (β) of one-dimensional Ising ferromagnetism was calculated for one-break configurations. In the limit of the applied magnetic field (H) approaches zero and the number of spins (N) approach infinity, the non – zero magnetization per particle was obtained using Fe, Ni, CrBr₃ and EuS materials as case studies.

The calculated values of magnetic critical point exponent (β) for Fe, Ni, CrBr₃ and EuS at N = 100 are 0.340 ± 0.042 ; 0.420 ± 0.070 ; 0.368 ± 0.005 and 0.330 ± 0.015 respectively.

According to Stanely [13], the range of values for magnetic exponent (β) is 0.3 – 0.5, which is in agreement with the results obtained. The experimental values of critical point exponent (β) of ferromagnetic is presented in Table 10; and is adopted from Itzykson and Drouffe [10].

KEYWORDS: Magnetic Critical Point Exponent, Magnetism, Ferromagnetic Materials

1.0 INTRODUCTION

Magnetism is an important and interesting concept in solid state physics. In fact all materials; insulators, semiconductors and conductors (metals) exhibit the phenomenon of magnetism. Magnetism can be classified as diamagnetism, paramagnetism and ferromagnetism.

Diamagnetic materials possess no net magnetic moments of their own origin. They do not possess magnetization in the absence of an applied magnetic field. When an external magnetic field is applied to such materials their atoms acquire magnetic moments whose direction is opposite to that of the applied field [1]. Paramagnetic materials are made up of atoms that possess their own magnetic moments, which are aligned in different directions. In such a configuration the material does not possess net macroscopic magnetization. When an external field is applied, the magnetic moments align in a definite direction.

Ferromagnetic materials have atoms possessing atomic moments that are aligned microscopically in particular directions. In such a way, different portions of the ferromagnetic materials have net magnetization per unit volume. Application of external magnetic field also strengthens the magnetization produced in such materials [2]. They are able to retain a substantial amount of magnetization. Of most practical and industrial applications are materials exhibiting ferromagnetism due to their ability to produce magnetization even in the absence of an applied field. Ferromagnetism are found useful as applications in electronic devices such as magnetic tapes, digital computer memories and in ferrite microwave devices.

According to Wolf [3], in order to identify materials with an Ising – like microscopic Hamiltonian, one needs to understand the behaviour of individual magnetic ions in a crystalline environment. The basis for this understanding comes from the early work of Van Vleck, as defined with the advent of paramagnetic resonance in the 1950's and the introduction of the spin Hamiltonian [4]. Medelung [5] identified that the interactions in magnetism have been explained by various theories and models. One of these models is the Ising Model, which could account for observed phenomena in 1 – D, 2 – D and 3 – D systems. A very important concept in ferromagnetic material is the issue of phase transition. Ferromagnets have spontaneous magnetization only at temperatures below a characteristic temperature known as the Curie point. Above the Curie point, the spontaneous magnetization ceases and the material become paramagnetic. The Curie point is a feature of all ferromagnetic materials and is constant for different materials. The thermodynamical quantities of the material also change on crossing the Curie point. Experimental results and detailed calculations have shown how the thermodynamic properties behave when approaching the Curie point [6].

The study of critical phenomena both experimentally and theoretically is the determination of the asymptotic law governing the approach to a critical point. The focus of this paper is on the calculation of magnetic exponent (β) using the 1 – D Ising model over one break. The order parameter (m) is a measure of the degree to which the magnetic moments are aligned throughout the crystal and is called the zero-field magnetization.

2.0 THEORETICAL BACKGROUND CONSIDERATION

2.1 Ising Model

The Ising model is an important paradigm because it captures the physics of several physical systems with short-range interactions. The standard model is a system of spins on a lattice with nearest neighbour interactions.

$$H = - \sum_{ij} I_{ij} S_i S_j - H \sum_{i=1}^N S_i \quad (2)$$

$$\text{Where } I_{ij} = \frac{J}{|i - j|^2} \quad (3)$$

Equation 1 is a one dimensional ferromagnetic model; where H is the external magnetic field; I_{ij} is the interaction energy between neighbouring spins; $S_i S_j$ are the spins situated at regular lattice sites; H represents an external magnetic field which applies to all spins on the lattice. Both S_i and S_j can independently assume either of the two values of H , $+1$ and -1 . The constant J in equation 2 is called the coupling constant, representing the interaction between nearest neighbours. It is also called the exchange parameter or effective interaction strength. $|i - j|$ is the distance between spin sites i and j .

In the present discussion we concentrate on the simplest version of the model, with positive coupling constant J between nearest neighbours only, and homogeneous external field H . Also take the spins values $s_i = \pm 1$, that is, dealing with spin – $\frac{1}{2}$ particles. Both s_i and s_j can independently assume either of the two values of $+1$ and -1 . Our objective is to determine the stable phases of the system in the thermodynamic limit, that is, when the number of spin $N \rightarrow \infty$ and the external field $H \rightarrow 0$. In this sense, the physics of the model is dominated by the interactions between nearest neighbours rather than by the presence of the external field; the latter only breaks the symmetry between spins pointing up (along the field) or down (opposite to the field). The phases of the system are characterized by the average value m of the total magnetization M at a given temperature:

$$m(T) = \overline{M}, \quad M = \sum_i S_i \tag{4}$$

From the expression for the free energy of the system at temperature T

$$F = E - TS = \overline{M} - TS \tag{5}$$

There are two possibilities; first, that the stable phase of the system has average magnetization, which is called the disordered phase; this phase, is favoured by the entropy term. Second, that the stable phase of the system has non-zero average magnetization, called the ordered phase; this phase is favoured by the internal energy term.

In one dimension, the nearest neighbour Ising model does not exhibit a phase transition at finite temperature. This model has no time dependent dynamics. In this respect, it is unlike the gas model and quantum mechanical models, in which the Hamiltonian function or operator determines the equation of motion.

The equilibrium properties of this model can be derived by the method of fixed temperature. The solution to the zero – field, $H = 0$ case is considered. If the temperature $T = 0$, then either all s_i are + 1 or all s_j are –1 so that H is a minimum, with a value

$$E(T = 0) = -J(N - 1) \tag{6}$$

When $T > 0$, then some of the s_i will be +1 and the others –1. The boundary between +1 region and the –1 region is called the partition point. At $T = 0$, there is no partition point and at low temperatures, the partition points are few. The energy of each partition point is 2J. This model is then transformed to a model of a gas of partition points. The number of partition points is not constant, so its chemical potential is zero.

2.3 The Magnetic Critical Point Exponent (β)

Critical point exponents describe the behaviour near the critical point of the various quantities of interest at phase transition. Examples are the exponents that describe: pressure, heat capacity, susceptibility, magnetization, energy and others. Among the critical exponents for magnetic systems, are those for specific heat, α ; magnetization, β ; isothermal susceptibility, γ ; and correlation length, ν . These are not all independent, and it is possible to derive an inequality such as: $\alpha + 2\beta + \gamma \geq 2$; which was given by Yeomans [8]. According to Tian and Gui [9] when ferromagnetic system is considered α is the critical exponent of susceptibility; β is the critical exponent of order parameter; γ is the critical exponent of the specific heat; and δ is the critical exponent of the magnetic field coupled with order parameter. Their experimental values are stated as follows; $\alpha = 0.104 \pm 0.003$; $\beta = 0.325$; $\gamma = 1.23$ and $\delta = 5.2 \pm 0.15$

Tian and Gui [9] also stated that experimental physicists discovered that the critical exponents about the thermodynamic quantities in various phase transitions satisfy the same scaling laws as:

$$\alpha = 2\beta + \delta \approx 2 \quad \gamma \approx \beta (\delta - 1) \tag{7}$$

According to Itzykson and Dronffe [10], the six commonly recognized critical points exponents are: $\alpha, \beta, \gamma, \delta, \eta$ and ν . The definitions of the first four are given as:

$$C_H \approx |t|^{-\alpha} \text{ when } H = 0 \quad M \approx |t|^\beta \text{ when } H = 0$$

$$\chi \approx |t|^{-\gamma} \quad \text{when } H = 0 \quad H \approx M^\delta \quad \text{when } H = 0$$

Critical systems have at least one order parameter. The order parameter is some quantity that takes on two values; one above criticality and one below. For the Ising system the order parameter is the one-dimensional magnetization of the system.

Clearly at low temperatures in the Ising model, all spins will try to align in the same direction. However at higher temperature, thermal fluctuations will tend to randomize spin orientation. Experimentally it is known that ferromagnetic possess a critical temperature at which the total magnetization of the system differs from zero. Temperatures below the critical temperature have a finite magnetization whereas temperatures above the critical point have zero magnetization. Also experimentally, it is known that critical systems will have nearly identical critical exponents if they have the same physical dimension as well as the same dimension of the order parameter. In this way, all three-dimensional ferromagnetism are the same. The ferromagnetism, which are characterized by the existence of a spontaneous magnetization, are given by:

$$m_o(t) = \lim_{H \rightarrow 0} m(H, T) \quad (8)$$

$$\text{The order parameter varies as } (-\epsilon)^\beta \text{ where } \epsilon \equiv \frac{T - T_c}{T_c}$$

$$\text{The magnetic exponent } (\beta) \text{ is also define as: } m_o(T) \approx (T - T_c)^\beta \quad (9)$$

That is when the external field H vanishes; the magnetization M below T_c is a decreasing function of the T and vanishes at T_c . A more natural definition of the critical point exponent (β) is defined as:

$$\beta = \lim_{\epsilon \rightarrow 0} \frac{\ln m_o(T)}{\ln(-\epsilon)} \quad (10)$$

Critical point exponents are frequently determined by measuring the slopes of log – log plots of experimental data, since l¹ Hospital rule with the equation above implies that

$$\beta \equiv \frac{d(\ln m)}{d \ln(-\epsilon)} \quad (11)$$

2.4 Partition Function Z and Magnetization M

The partition function is an unrestricted sum of Boltzmann factors over all accessible states, irrespective of their energy. Hence it is generally easier to derive statistical thermo-dynamical results using the partition function.

Classical and quantum statistical mechanics show that all the thermo-dynamical properties of a system can be expressed in terms of $\ln Z$ and its partial derivatives.

The magnetic partition function $Z(H, T)$ for Ising model is given as:

$$Z = \sum_{S_1} \sum_{S_2} \dots \sum_{S_N} e^{-tH} \quad (12)$$

where each s_i ranges independently over the values ± 1 and there are 2^N terms in the summation; also $t = 1/ k_B T$. The thermodynamic function as related to magnetization is given as:

$$\text{Magnetization } m(H, T) = -\frac{\partial}{\partial H} \left[\frac{F(H, T)}{KT} \right] = \left\langle \sum_{i=1}^N S_i \right\rangle \tag{13}$$

Where $\langle \rangle$ denotes ensemble average. The quantity $m(0, T)$ is called the spontaneous magnetization. When it is non-zero the system is said to be ferromagnetic, while when it is zero, the system is said to be paramagnetic. The magnetization m , plays the role of the order parameter, which determines the nature of the phase above and below the critical temperature. While the critical exponents describe the behaviour of various physical quantities close to the critical temperature.

3.0 NUMERICAL COMPUTATION

The spins of the one-dimensional ferromagnetic is as shown in figure 1, which is a simple one dimensional arrow shape, capable of assuming two discrete orientations via: +1 for spin up and -1 for spin down.

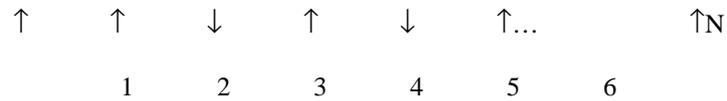


Figure 3.1: Schematic Diagram of the Spins Using Arrows

3.1 Computation for N = 3 Spins

The schematic diagram representing each state of the system for $N = 3$ is shown in Table 1. From the schematic diagram there are 2^3 (i.e. 2^N) configuration states of the system. The partition function is given as the sum of the partition functions of all the configuration states. There are two nearest neighbours to the s_i under consideration.

$S_i = + 1$	$S_i = \pm 1$	$s_i = - 1$	$s_i = \pm 1$
Orientation	Hamiltonian	Orientation	Hamiltonian
$\uparrow\uparrow\uparrow$	$\frac{-9J}{4} + 3H$	$\downarrow\downarrow\downarrow$	$\frac{-9J}{4} - 3H$
$\uparrow\downarrow\uparrow$	$\frac{-7J}{4} + H$	$\downarrow\uparrow\downarrow$	$\frac{-7J}{4} - H$
$\uparrow\uparrow\downarrow$	$\frac{-J}{4} + H$	$\downarrow\downarrow\uparrow$	$\frac{-J}{4} - H$
$\downarrow\downarrow\downarrow$	$\frac{-J}{4} - H$	$\downarrow\uparrow\uparrow$	$\frac{-J}{4} + H$

Figure 3.2: Schematic Arrowed Spin Diagram for N = 3

$$Z = \left[\exp \left(\frac{9Jt}{4} \right) \right] \left[\exp(3Ht) + \exp(-3Ht) \right]$$

$$\begin{aligned}
& + \left[\exp \left(\frac{7Jt}{4} \right) \right] [\exp (Ht) + \exp (-Ht)] \\
& + 2 \left[\exp \left(\frac{-Jt}{4} \right) \right] [\exp (Ht) + \exp (-Ht)] + \dots
\end{aligned} \tag{14}$$

$$\begin{aligned}
Z = & [\exp (3Ht)] \left[\exp \left(\frac{9Jt}{4} \right) \right] + [\exp (-2Ht)] \left\{ 2 \exp \left(\frac{-Jt}{4} \right) + \exp \left(\frac{-7Jt}{4} \right) \right\} \\
& + [\exp (-4Ht)] \left\{ \exp \left(\frac{-Jt}{4} \right) + \exp \left(\frac{-7Jt}{4} \right) \right\} + \left[\exp \left(\frac{9Jt}{4} \right) \right] [\exp (-6Ht)] \tag{14b}
\end{aligned}$$

Putting $Z = \exp (-2Ht)$ (15)

$$\frac{\partial Z}{\partial Ht} = -2 \exp (-2Ht) = -2Z \tag{16}$$

Equation (14b) becomes: $Z = [\exp (3Ht)] \{C_0 + C_1Z + C_2Z^2 + C_3Z^3\}$ (17)

where $C_0 = C_3 = \exp \left(\frac{9Jt}{4} \right)$ and $C_1 = C_2 = 2 \exp \left(\frac{-Jt}{4} \right) + \exp \left(\frac{-7Jt}{4} \right)$

$$\ln Z = 3Ht + \ln [C_0 + C_1Z + C_2Z^2 + C_3Z^3 + \dots] \tag{18}$$

$$\text{Total magnetization } m \equiv \frac{\partial \ln Z}{\partial (Ht)} = \frac{\partial z}{\partial (Ht)} = \frac{\partial \ln Z}{\partial z}$$

Magnetization per particle is $m = M/N$ and for this case

$$m = \frac{1}{3} M = \frac{\partial \ln Z}{\partial z} \tag{19}$$

Using equation (13) and equation (14)

$$m = \frac{1}{3} \frac{\partial \ln Z}{\partial z} = 1 - \frac{2}{3} Z \left[\frac{C_1 + 2C_2Z + 3C_3Z^2}{C_0 + C_1Z + C_2Z^2 + C_3Z^3} \right] \tag{20}$$

In the limit $H \rightarrow 0$ $z \rightarrow 1$ and the magnetization per particle becomes

$$m \equiv \lim_{H \rightarrow 0} \frac{\partial \ln z}{3 \partial (Ht)} = \lim_{z \rightarrow 1} \frac{\partial \ln z}{3 \partial (Ht)}$$

Spontaneous magnetization per particle of equation (15) is

$$m = 1 - \frac{2}{3} z \left[\frac{C_1 + 2C_2 + 3C_3}{C_0 + C_1 + C_2 + C_3} \right] \tag{21}$$

$$m = \frac{1}{(2N+1)} \frac{\partial \ln Z}{\partial z} \frac{\sum_{j=-N}^N \ell^{2Jm^2 R_j} \cdot \frac{(2j+1)}{(2N+1)} \cdot \ell^{(2j+1)N}}{\sum_{j=-N}^N \ell^{2Jm^2 R_j} \cdot \ell H} \quad (27)$$

and in the limit $H \rightarrow 0$

$$m \equiv \frac{\sum_{j=-N}^N \ell^{2Jm^2 R_j} \cdot \frac{(2j+1)}{(2N+1)}}{\sum_{j=-N}^N \ell^{2Jm^2 R_j}} \quad (28)$$

From equation (25)

$$R_j = R_0 + \ln \left[1 - \left(\frac{j}{N+1} \right)^2 \right] = R_0 + \ln(1 + y_j) \quad (29)$$

$$\text{Where } y_j \equiv \left(\frac{j}{N+1} \right)^2 \quad (30)$$

In equation (28), putting

$$X = 2Jm^2 \quad (31)$$

$$\text{Then } m = q \sqrt{X} \quad (32)$$

$$\text{Where } q^2 = \frac{1}{2Jt} = \frac{K_B T}{2J} \quad (33)$$

Equation (28) can be rewritten as

$$m = \frac{\sum_{j=-N}^N \ell^{[R_0 + \ln(1-y_j)]^X} \cdot \frac{(2j+1)}{(2N+1)}}{\sum_{j=-N}^N \ell^{[R_0 + \ln(1-y_j)]^X}} \quad (34)$$

$$m^2 = \frac{\sum_{j=-N}^N \left(\frac{(2j+1)}{(2N+1)} \right)^2 (1-y_j)^X}{\sum_{j=-N}^N (1-y_j)^X} \neq 0 \quad \text{if } j \geq 0 \quad (35)$$

The required solutions are the values of m that will satisfy both equations (32) and (35). A computer program in C++ language was used to provide the solutions.

4.0 RESULTS AND DISCUSSIONS

From section 3 above the following can be deduced:

- As X increases, t increases and T decreases
- As q increases t decreases and T increases

$$\text{That is } q^2 \propto \frac{1}{t} \propto T \tag{36}$$

According to Ojo [12] the critical temperature for which the critical magnetization $m_c \neq 0$ is given by:

$$K_B T_c = 1.47J \tag{37}$$

Using equations 33 and 37

$$q_c^2 = \frac{K_B T_c}{2J} = 0.735 \tag{38}$$

Using equation (36) together with the definition of exponents:

$$\epsilon = \frac{q^2 - q_c^2}{q_c^2} \approx \frac{T - T_c}{T_c} \tag{39}$$

$$\beta = \frac{d \ln m}{d \ln (-\epsilon)} \tag{40}$$

β is calculated as the shape of the graph $d \ln (m)$ against $d \ln (-\epsilon)$; which is the same as graph of $(\ln X)$ against $|\ln |\epsilon||$. Results for different elements for $n = 100$ and $N = 1000$ are presented in Tables 1 – 8.

Table 4.1: Results of Fe when N = 100

T	X	$ \epsilon $	$ \ln X $	$ \ln \epsilon $
100	0.0003480	0.9041	7.9632	0.1008
200	0.0001740	0.8082	8.6563	0.2129
300	0.0001160	0.7124	9.0618	0.3392
400	0.0000870	0.6165	9.3494	0.4837
500	0.00000696	0.5206	9.5726	0.6527
600	5.8009×10^{-5}	0.4247	9.7549	0.8563
700	4.9722×10^{-5}	0.3289	9.9091	1.1121
800	4.3507×10^{-5}	0.2330	10.0426	1.4568
900	3.8673×10^{-5}	0.1371	10.1604	1.9870
1000	3.4805×10^{-5}	0.0412	10.2657	3.1887

Table 4.2: Results of Fe when N = 1000

T	X	$ \epsilon $	$ \ln X $	$ \ln \epsilon $
100	3.543×10^{-6}	0.9041	12.5506	0.1008
200	1.771×10^{-6}	0.8082	13.2438	0.2129
300	1.181×10^{-6}	0.7124	13.6492	0.3392
400	8.860×10^{-7}	0.6165	13.9369	0.4837
500	7.090×10^{-7}	0.5206	14.1601	0.6527
600	5.900×10^{-7}	0.4247	14.3424	0.8563
700	5.060×10^{-7}	0.3289	14.4965	1.1121
800	4.430×10^{-7}	0.2330	14.6301	1.4568
900	3.940×10^{-7}	0.1371	14.7479	1.9870
1000	3.540×10^{-7}	0.0412	14.8532	3.1887

Table 4.3: Results of Ni when N = 100

T	X	$ \epsilon $	$ \ln X $	$ \ln \epsilon $
100	2.1112×10^{-4}	0.8406	8.4631	0.1737
200	1.0556×10^{-4}	0.6811	9.1562	0.3840
300	7.0374×10^{-5}	0.5217	9.5617	0.6507
400	5.2780×10^{-5}	0.3624	9.8494	1.0154
500	4.2224×10^{-5}	0.2028	10.0725	1.5955
600	3.5187×10^{-5}	0.0434	10.2548	3.1380

Table 4.4: Results of Ni when N = 1000

T	X	$ \epsilon $	$ \ln X $	$ \ln \epsilon $
100	2.130×10^{-6}	0.8406	13.0592	0.1737
200	1.065×10^{-6}	0.6811	13.7524	0.3840
300	7.100×10^{-7}	0.5217	14.1578	0.6507
400	5.330×10^{-7}	0.3624	14.4455	1.0154
500	4.260×10^{-7}	0.2028	14.6687	1.5955
600	3.550×10^{-7}	0.0434	14.8510	3.1380

Table 4.5: Results of CrBr₃ when N = 100

T	X	$ \epsilon $	$ \ln X $	$ \ln \epsilon $
9	1.2178×10^{-4}	0.7236	9.0133	0.3235
12	9.1333×10^{-5}	0.6314	9.3010	0.4597
15	7.3066×10^{-5}	0.5393	9.5241	0.6175
18	6.0889×10^{-5}	0.4472	9.7065	0.8048
21	5.219×10^{-5}	0.3550	9.8606	1.0355
24	4.566×10^{-5}	0.2629	9.9941	1.3360
27	4.0592×10^{-5}	0.1708	10.1119	1.7675
30	3.6533×10^{-5}	0.0786	10.2173	2.5431

Table 4.6: Results of CrBr₃ when N = 1000

T	X	ϵ	ln X	ln ϵ
9	1.229×10^{-6}	0.7236	13.6095	0.3235
12	9.220×10^{-7}	0.6314	13.8971	0.4597
15	7.370×10^{-7}	0.5393	14.1203	0.6175
18	6.144×10^{-7}	0.4472	14.3026	0.8048
21	5.270×10^{-7}	0.3550	14.4568	1.0355
24	4.617×10^{-7}	0.2629	14.5903	1.3360
27	4.100×10^{-7}	0.1708	14.7081	1.7675
30	3.690×10^{-7}	0.0786	14.8134	2.5431

Table 4.7: Results of EuS when N = 100

T	X	ϵ	ln X	ln ϵ
6	9.2567×10^{-5}	0.6364	9.2876	0.4520
7	7.9340×10^{-5}	0.5758	9.4417	0.5521
8	6.9426×10^{-5}	0.5152	9.5253	0.6633
9	6.1712×10^{-5}	0.4545	9.6930	0.7885
10	5.5540×10^{-5}	0.3939	9.7984	0.9316
11	5.0491×10^{-5}	0.3333	9.8937	1.0986
12	4.6284×10^{-5}	0.2727	9.9807	1.2993
13	4.2723×10^{-5}	0.2121	10.0608	1.5506
14	3.9672×10^{-5}	0.1515	10.1349	1.8871
15	3.7027×10^{-5}	0.0909	10.2039	2.3979
16	3.4713×10^{-5}	0.0303	10.2684	3.965

Table 4.8: Results of EuS when N = 1000

T	X	ϵ	ln X	ln ϵ
6	9.340×10^{-7}	0.6364	13.8837	0.4520
7	8.010×10^{-7}	0.5758	14.0379	0.5521
8	7.010×10^{-7}	0.5152	14.1714	0.6633
9	6.230×10^{-7}	0.4545	14.2892	0.7885
10	5.600×10^{-7}	0.3939	14.3945	0.9316
11	5.090×10^{-7}	0.3333	14.4898	1.0986
12	4.670×10^{-7}	0.2727	14.5769	1.2993
13	4.310×10^{-7}	0.2121	14.6569	1.5506
14	4.000×10^{-7}	0.1515	14.7310	1.8871
15	3.740×10^{-7}	0.0909	14.8000	2.3979
16	3.500×10^{-7}	0.0303	14.8645	3.965

The tables obtained from the theoretical calculations were used to plot the following graphs, with the fitted equations showing coefficients of determination ranging from 0.9 to 1.0

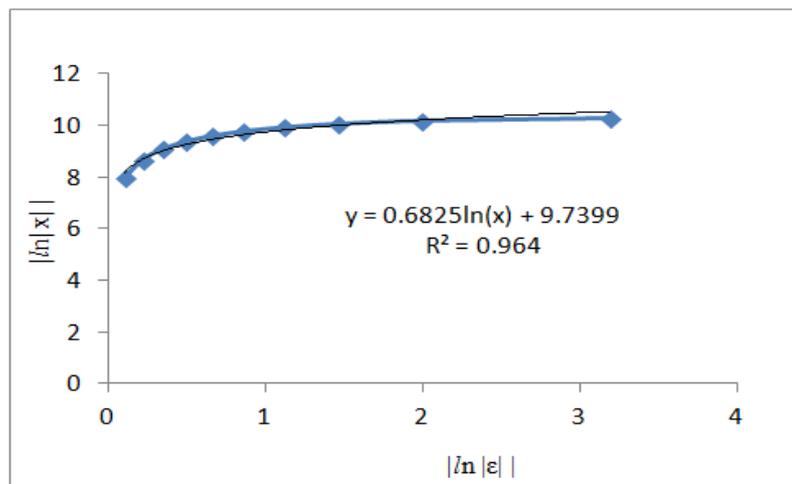


Figure 4.1: Graph of $|\ln|x||$ Against $|\ln|\varepsilon||$ for Fe when $N = 100$

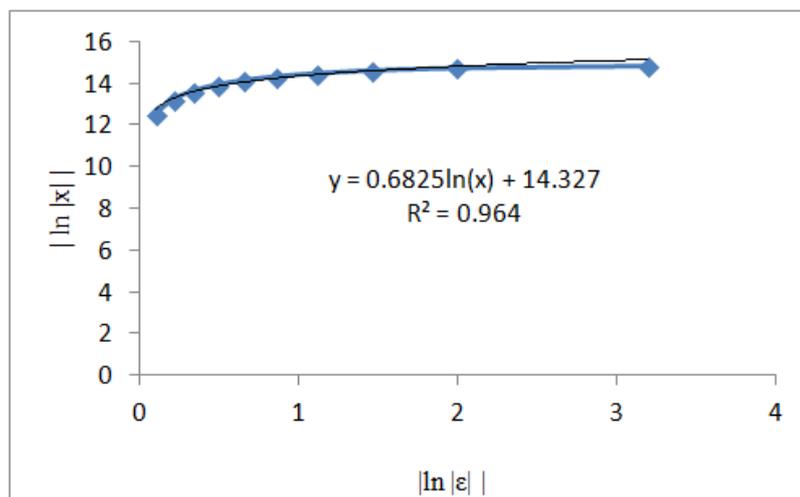


Figure 4.2: Graph of $|\ln|x||$ against $|\ln|\varepsilon||$ for Fe when $N = 1000$

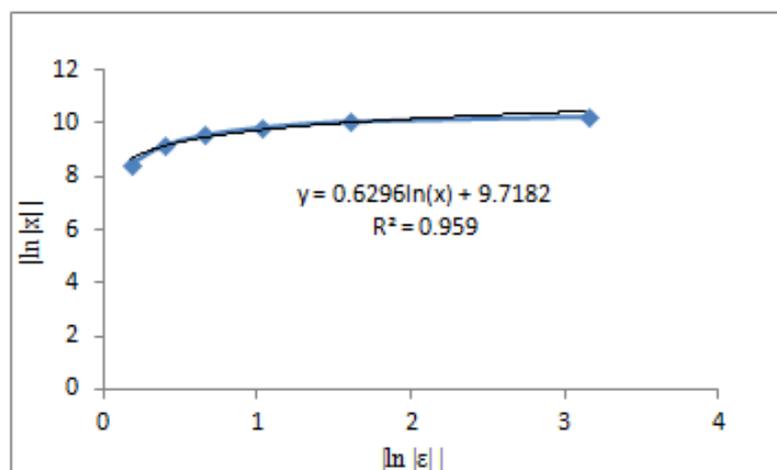


Figure 4.3: Graph of $|\ln|x||$ against $|\ln|\varepsilon||$ for Ni when $N = 100$

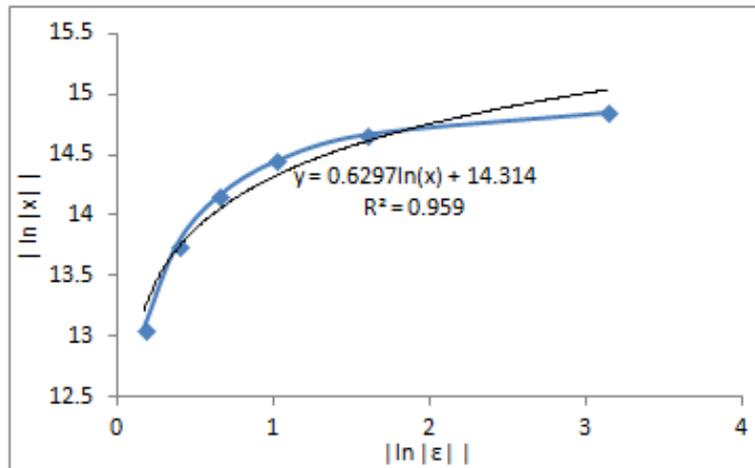


Figure 4.4: Graph of $|\ln|x||$ against $|\ln|\epsilon||$ for Ni when $N = 1000$

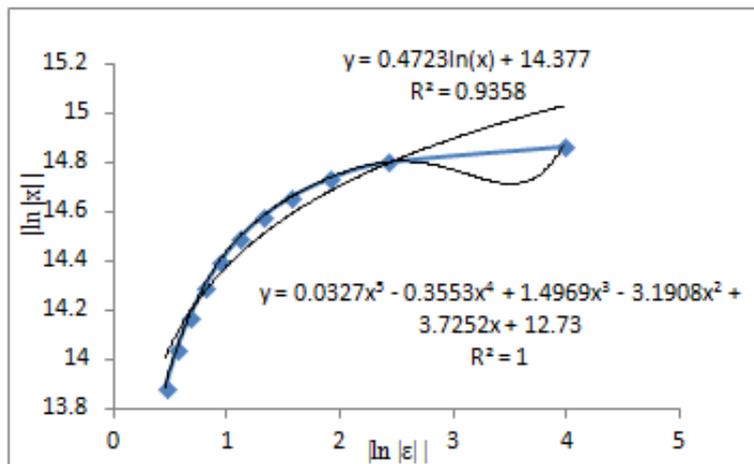


Figure 4.5: Graph of $|\ln|x||$ against $|\ln|\epsilon||$ for CrBr_3 when $N = 100$

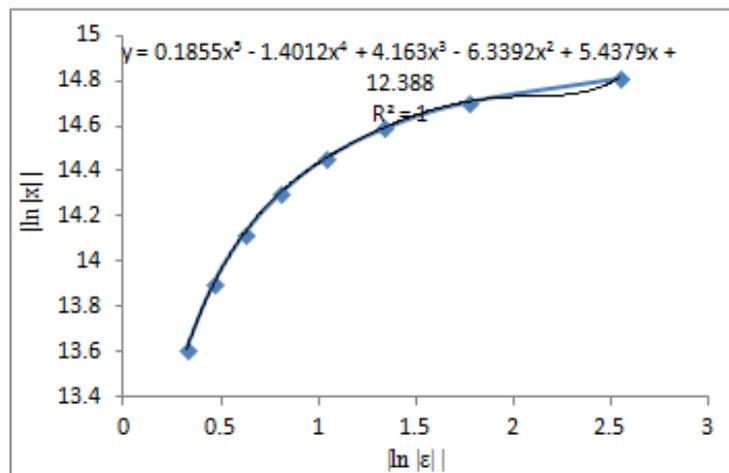


Figure 4.6: Graph of $|\ln|x||$ against $|\ln|\epsilon||$ for CrBr_3 when $N = 1000$

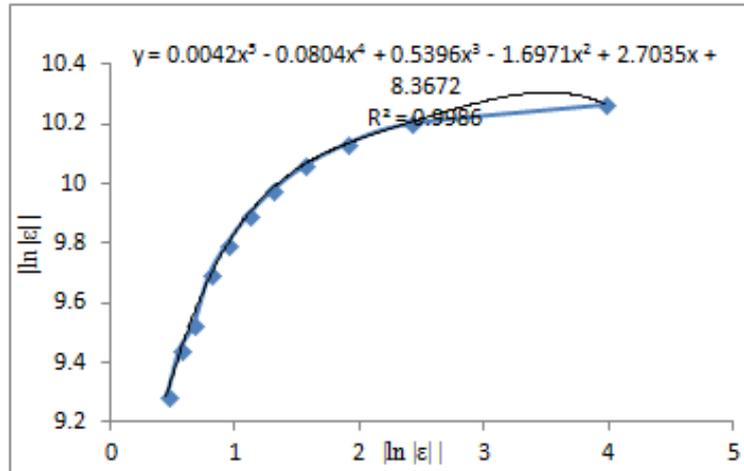


Figure 4.7: Graph of $|\ln|x||$ against $|\ln|\epsilon||$ for EuS when $N = 100$

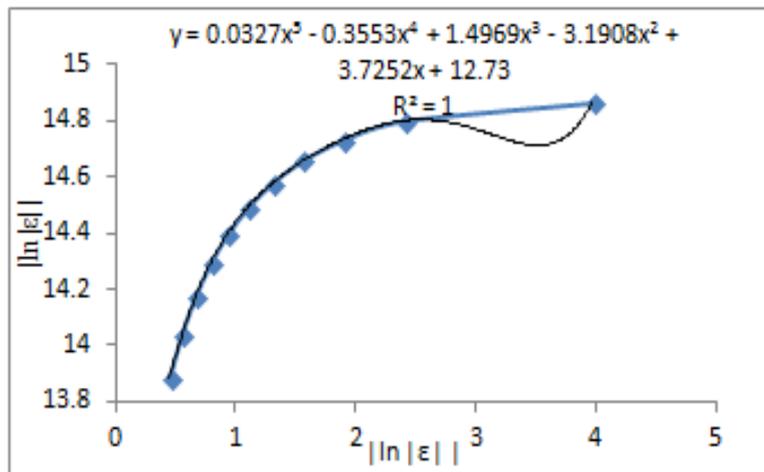


Figure 4.8: Graph of $|\ln|x||$ against $|\ln|\epsilon||$ for EuS when $N = 1000$

5.0 SUMMARY AND CONCLUSIONS

Table 5.1: Summary of the Results Obtained

Material	Curie Temperature (K)	Experimental values of β	β - when $N = 100$	β - when $N = 1000$	Average
Fe	1043	0.340 ± 0.400	0.320	0.330	0.325
Ni	627.2	0.422 ± 0.070	0.450	0.370	0.410
CrBr ₃	32.56	0.368 ± 0.005	0.375	0.378	0.376
EuS	16.50	0.330 ± 0.015	0.350	0.345	0.347

According to Stanely [13], the range of values for magnetic exponent (β) is 0.3 – 0.5, which is in agreement with the results obtained. The experimental values of critical point exponent (β) of ferromagnetic is presented in Table 10; and is adopted from Itzykson and Drouffe [10].

Table 10: Experimental Values adopted from Itzykson and Drouffe.

EXPT	MFT	ISING – D	HEIS
0.32 – 0.39	0.50	0.31	0.30

Where EXPT is the experimental values from a variety of systems; MFT is the Mean Field theory values; ISING is the Ising Model values in the D – dimension array used; and HEIS is the values for Classical Heisenberg Model

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