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ON THE USE OF ABOODA DECOMPOSITION METHOD FOR SOLVINGHIGHER-ORDER INTEGRA-DIFFERENTIAL EQUATIONS

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ABSTRACT

In this research, higher-order Volterra-Fredholm integro-differential equations were solved using the Aboodh decomposition technique (ADM).

INTRODUCTION

This research focuses on significant issues in engineering sciences that involve both integral and differential elements.

The article's goal is to improve analysis of the modified Aboodh decomposition method (ADM), which will help to solve the higher-order IDE of Volterra's form.

$$y^{(n)} = f(z) + \lambda_1 + \int_a^b k_1(z,\tau)W_1\big(y(\tau)\big)\big)d\tau + + \lambda_2 + \int_a^b k_1(z,\tau)W_1\big(y(\tau)\big)\big)d\tau \quad (1)$$

Lead to

$$y^{(m)}(a) = y_m m = 0,1,2,\dots,n-1 \quad (2)$$

$a, b, \lambda_1, \lambda_2, y_m$ are constant values, $f(z).\,k_1(z,\tau), k_2(z,\tau)$ are given forms with

Appropriate interval derivatives$a \le z \le \tau \le b, W_1\big(y(\tau)\big), and\ W_2 y(\tau))$are non linear function sin$y(\tau)_{.2}$.

Abooda Transformation Method (ATM) with Their Properties

The Abooda transform (AT) of $f(\tau)\epsilon\Re, \forall\ \tau \ge 0$

$$\boldsymbol{A}[\boldsymbol{f}(\boldsymbol{\tau})] = \boldsymbol{F}(\boldsymbol{\omega}) = \frac{1}{v}\int_a^\infty \boldsymbol{f}(\boldsymbol{\tau})\boldsymbol{e}^{-\tau v}\,\boldsymbol{d\tau} \quad \boldsymbol{h_1} \le \frac{1}{v} \le h_2 \quad (3)$$

Since $A[.]$ is the operator of Aboodh transform and the set R is defined as

$$\Re = \{f(\tau)|\exists C, h_1, h_2 > 0, \big|\boldsymbol{f}(\boldsymbol{\tau}) < C\boldsymbol{e}^{\tau/h_n}, \boldsymbol{if}\ \boldsymbol{\tau} \in (-\mathbf{1})^{\boldsymbol{n}} \times [\mathbf{0},\infty)\big|\}, \quad (4)$$

can be finite or infinite, and $\boldsymbol{C}$ is a real finite number. $\boldsymbol{A}[\boldsymbol{f}(\boldsymbol{\tau})] = \boldsymbol{F}(\boldsymbol{v})$ then $\boldsymbol{f}(\boldsymbol{\tau})$ is called the inverse transform of Abooda to $\boldsymbol{F}(\boldsymbol{\omega})$and denoted by

$\boldsymbol{E}[\boldsymbol{f}(\boldsymbol{\tau})] = \boldsymbol{A}^{-1}[\boldsymbol{F}(\boldsymbol{v})]$where $\boldsymbol{A}^{-1}$ is the operator of inverse Aboodh transform. Table 1 shows the transformation of Aboodh for some basic functions [22].

Special properties of the Aboodh transform are as follows.

Theorem 1 (linearity, see [23]). If $F_1(\omega)$and $F_2(\omega)$ are Aboodh transform (ET) of $F_1(\tau)$ and $F_2(\tau)$respectively, then

$$A\{\ rf_1(\tau) + \ s\,f_2(\tau)\} = \{\ rf_1(\tau)\} + \{s\,f_2(\tau)\} = \ rF_1(\tau) + \ s\,F_2(\tau) \quad \textbf{(5)}$$

r, s are constants.

Theorem 2

If the transform of Aboodh for $f(\tau)$given by $A[f(\tau)] = F(\omega)$then $A[f'^{(\tau)}(\tau)] = v^2F(v) - \frac{1}{v}f(0)$

$$A[f''^{(\tau)}(\tau)] = v^2F(v) - f(0) - \frac{1}{v}f'(0) \quad (6)$$

$$A[f^{(n)}(\tau)] = v^nF(v) - \sum_{k=0}^{n-1}\left(\frac{1}{v}\right)^{2-n+k} f^{(k)}(0).$$

Theorem 3

If $f(\tau) \in \Re$ with Aboodh transform $F(v)$, then

$$A[e^{a\tau}f(\tau)] = \frac{v}{v-a}F\left[\frac{1}{v-a}\right] \quad \textbf{(7)}$$

Theorem 4

If $f_1(\tau)$ and $f_2(\tau)$are functions with Aboodh transform $F_1(\tau)$ and $F_2(\tau)$, then the convolution of $f_1$and f_2 is

$f(\tau)$	$A[f(\tau)] = F(v)$
1	$\frac{1}{v^2}$
r	$\frac{1}{v^3}$
r^2	$\frac{2!}{v^4}$
$r^n n \in N$	$\frac{n!}{v^{n+2}}$
$e^{a\tau}$	$\frac{1}{v^2 - av}$
$\sin(a\tau)$	$\frac{a}{v^3 + a^2v}$
$\cos(a\tau)$	$\frac{1}{v^2 + a^2}$
$\sinh(a\tau)$	$\frac{a}{v^3 - a^2v}$
$\cosh(a\tau)$	$\frac{1}{v^2 - a^2}$
$\tau e^{ae^{at}}$	$\frac{1}{v(v-a)^2}$

$$(f_1 * f_2)(\tau) = \int_a^\tau f_1(x)f_2(\tau - x)dx \quad \textbf{(8)}$$

$$A[f_1 * f]_2 = \frac{1}{\omega}F_1(\omega)F_2(\omega) \quad \textbf{(9)}$$

Theorem 5

Let F (ω) denote to the transform of Aboodh for $\boldsymbol{f(\tau)}$,then the definite integral of $\boldsymbol{f(\tau)}$, $L(\tau) = \int_0^\tau f(\tau)d\tau$,(10) has transform of Aboodh as

$$L(\tau) = \frac{\mathbf{1}}{v}F(v). \tag{11}$$

THE PROPOSED TECHNIQUE (ADM)

The suggested approach (ADM) is as follows: Assume that (1) and (2) have a distinct solution that is sufficiently differentiable (z). To provide an approximation for the high order integral-differential equations, we employ the Abooda decomposition technique.

Applying the ATM to (1)'s two sides, we obtain

$$\boldsymbol{A}[\boldsymbol{y}^{(n)}(\boldsymbol{z})] = \boldsymbol{A}[\boldsymbol{f}(\boldsymbol{z})] = \boldsymbol{A}[\lambda_1 \int_{\boldsymbol{a}}^{\boldsymbol{z}} \boldsymbol{k_1}(\boldsymbol{z},\boldsymbol{\tau})\boldsymbol{W_1}\boldsymbol{y}(\boldsymbol{\tau})\boldsymbol{d\tau}] + \boldsymbol{A}[\lambda_2 \int_a^z k_2(z,\tau)W_2 y(\tau)d\tau] \tag{12}$$

Then

$$v^n A[y(z)] - \sum_{k=0}^{n-1}\left(\frac{1}{v}\right)^{2-n+k} y^{(k)}(0) = A[f(z)] + A[\lambda_1 \int_a^z k_1(z,\tau)W_1(y(\tau))d\tau] + A[\lambda_2 \int_a^z k_2(z,\tau)W_2(y(\tau))d\tau] \tag{13}$$

Solving for $A[y(z)]$

$$A[y(z)] = \omega^n \sum_{k=0}^{n-1}\left(\frac{1}{v}\right)^{2-n+k} y^{(k)}(0) + \left(\frac{1}{v}\right)^n A[f(z)] + \left(\frac{1}{v}\right)^n A[\lambda_1 \int_a^z k_1(z,\tau)W_1(y(\tau))d\tau] +$$
$$\left(\frac{1}{v}\right)^n A\left[\lambda_2 \int_a^b k_2(z,\tau)W_2(y(\tau))d\tau\right] 4) \qquad (14)$$

Now

$$y(z) = \sum_{j=0}^{\infty} y_j(z). \tag{15}$$

A [y (z)] will be obtained, Then

$$W_1(y) = \sum_{j=0}^{\infty} A_j, W_2(y) = \sum_{j=0}^{\infty} B_j, \tag{16}$$

where A_j and $B_j$are the Adomian polynomials.

Then we have

$$\boldsymbol{A}\left[\sum_{\boldsymbol{j=0}}^{\infty} \boldsymbol{y_j}(\boldsymbol{z})\right] = \boldsymbol{\omega^n} \sum_{\boldsymbol{k=0}}^{\boldsymbol{n-1}} \left(\frac{\boldsymbol{1}}{\boldsymbol{v}}\right)^{\boldsymbol{2-n+k}} \boldsymbol{y^{(k)}(0)} + \frac{\boldsymbol{1}}{\boldsymbol{v^n}}\boldsymbol{A}[\boldsymbol{f}(\boldsymbol{z})] + \frac{\boldsymbol{1}}{\boldsymbol{v^n}}\boldsymbol{A}\left[\lambda_1 \int_a^z k_1(z,\tau) \sum_{j=0}^{\infty} A_j \, d\tau\right]$$
$$+ \frac{\boldsymbol{1}}{\boldsymbol{v^n}}\boldsymbol{A}\left[\lambda_2 \int_a^b k_2(z,\tau) \sum_{j=0}^{\infty} B_j \, d\tau\right] \tag{17}$$

$$A[y_0(z)] = \frac{1}{v^n}\sum_{k=0}^{n-1}\left(\frac{1}{v}\right)^{2-n+k} y^{(k)}(0) + \frac{1}{v^n}A[f(z)] \tag{18}$$

Then

$$A\left[\sum_{j=0}^{\infty} y_{j+1}(z)\right] = \frac{1}{v^n}A\left[\lambda_1 \int_a^z k_1(z,\tau)\sum_{j=0}^{\infty} A_j\, d\tau\right] + \frac{1}{v^n}A\left[\lambda_2 \int_a^b k_2(z,\tau)\sum_{j=0}^{\infty} B_j\, d\tau\right] \tag{19}$$

applying the inverse Aboodh transform of (18) and (19)

$$y_0(z) = A^{-1}\left[\frac{1}{v^n}\sum_{k=0}^{n-1}\left(\frac{1}{v}\right)^{2-n+k} y^{(k)}(0)\right] + A^{-1}\left[\frac{1}{v^n}A[f(z)]\right] y_{j+1}(z = A^{-1}\left[\frac{1}{v^n}A\left[\lambda_1 \int_a^z k_1(z,\tau)\sum_{j=0}^{\infty} A_j\, d\tau\right]\right] + E^{-1}\left[\frac{1}{v^n}A\left[\lambda_2 \int_a^b k_2(z,\tau)\sum_{j=0}^{\infty} B_j\, d\tau\right]\right], j \geq 0 \tag{20}$$

NUMERICAL EXAMPLES

Example 1. The third order linear IDE of Volterra- Fredholm is

$$y'''(z) = -\frac{1}{2}z^2 + \int_0^z y(\tau)d\tau + \int_{-\pi}^{\pi} z y(\tau)d\tau \quad 0 \leq z \leq 1, \tag{21}$$

Lead to

$$y(0) = 1,$$

$$y'(0) = 1,$$

$$y''(0) = -1 \tag{22}$$

$$A[y'''(z)] = A\left[-\frac{1}{2}z^2\right] + A\left[\int_a^z y(\tau)d\tau\right] \tag{23}$$

from theorems 2 &5 we have

$$A[y(z)] = 1 + v - \frac{1}{v} - \frac{1}{v^4} + \frac{1}{v}A[y(z)] + \frac{1}{v^3}\int_{-\pi}^{\pi} y(\tau)d\tau \tag{24}$$

Then we have

$$A[y(z)] = \frac{v^2}{v^4-1} + \frac{v}{v^4-1} - \frac{1}{v^4-1} - \frac{1}{v^3(v^4-1)} + \frac{1}{v^2(v^4-1)}\int_{-\pi}^{\pi} y(\tau)d\tau \tag{25}$$

we get

$$A[y_0(z)] = \frac{v^2}{v^4-1} + \frac{v}{v^4-1} - \frac{1}{v^4-1} - \frac{1}{v^3(v^4-1)} A[y_{j+1}(z)] = \frac{1}{v^2(v^4-1)}\int_{-\pi}^{\pi} y(\tau)d\tau, \quad j = 0,1,2 \tag{26}$$

then

$$y_0(z) = z + \cos z,$$

$$y_1(z) = A^{-1}\left[\frac{1}{v^2(v^4-1)}\int_{-\pi}^{\pi} y_0(\tau)d\tau\right] = 0, \tag{27}$$

$$y_{J+1}(z) = A^{-1}\left[\frac{1}{v^2(v^4-1)}\int_{-\pi}^{\pi} y_j(\tau)d\tau\right] = 0, \ \forall j = 0,1,2, \ldots.$$

So, the solution is

$$y(z) = z + \cos z$$

Example 2

$$y^{(4)}(z) = -\frac{z^6}{30} - \frac{z^4}{6} - \frac{z^2}{2} + \frac{8}{3}z + \frac{3}{2} + \int_0^z (z-\tau) y^2(\tau) d\tau - 2\int_0^1 (z+\tau) y(\tau) d\tau \; 0 \le z \le 1, \quad (28)$$

With

$$y(0) = 1,$$

$$y'(0) = 0,$$

$$y''(0) = 2,$$

$$y'''(0) = 0. \quad (29)$$

we have

$$A[y^{(4)}(z)] =$$

$$A\left[-\frac{z^6}{30}\right] - A\left[\frac{z^4}{6}\right] - A\left[\frac{z^2}{2}\right] + A\left[\frac{8}{3}z\right] + A\left[\frac{3}{2}\right] + A\left[\int_0^z (z-\tau) y^2(\tau) d\tau\right] - A\left[2\int_0^1 (z+\tau) y(\tau) d\tau\right] \quad (30)$$

Then we obtained

$$v^4 A[y(z)] = v^2 + 2 - 24\frac{1}{v^8} - 4\frac{1}{v^6} - \frac{1}{v^4} + \frac{8}{3}\frac{1}{v^3} + \frac{3}{2}\frac{1}{v^2} + \frac{1}{v^2}A[y^2(\tau)] - 2A\left[\int_0^1 (z+\tau) y(\tau) d\tau\right] \quad (31)$$

Then

$$A[y(z)] = \frac{1}{v^2} + \frac{2}{v^4} - \frac{24}{v^{12}} - \frac{4}{v^{10}} - \frac{1}{v^8} + \frac{8}{3v^7} + \frac{3}{2v^6} + \frac{1}{v^6}A[y^2(\tau)] - \frac{2}{v^4}A\left[\int_0^1 (z+\tau) y(\tau) d\tau\right] \quad (32)$$

using the recursive relation, we have

$$A[y_0(z)] = \frac{1}{v^2} + \frac{2}{v^4} - \frac{24}{v^{12}} - \frac{4}{v^{10}} - \frac{1}{v^8} + \frac{8}{3v^7} + \frac{3}{2v^6},$$

$$A[y_{j+1}(z)] = \frac{1}{v^6}A[A_j(\tau)] - 2\omega^4 A\left[\int_0^1 (z+\tau) y(\tau) d\tau\right], \quad j \ge 0, \quad (33)$$

A_n' s are given by

$$A_0(\tau) = {y_0}^2,$$

$$A_1(\tau) = 2y_0 y_1,$$

$$A_2(\tau) = 2y_0 y_2 + {y_1}^2,$$

$$A_3(\tau) = 2y_0 y_3 + 2y_1 y_2,$$

We apply the inverse Aboodh transform; we have

$$y_0(z) = A^{-1}\left[\frac{1}{v^2} + \frac{2}{v^4} - \frac{24}{v^{12}} - \frac{4}{v^{10}} - \frac{1}{v^8} + \frac{8}{3v^7} + \frac{3}{2v^6},\right],$$

$$y_{j+1}(z) = A^{-1}\left[\frac{1}{v^6}A\left[A_j(\tau)\right]\right] - 2A^{-1}\left[\frac{1}{v^4}A\left[\int_0^1 (z+\tau)y(\tau)d\tau\right]\right],\ \ j \geq 0,$$

So

$$y_0(z) = 1 + z^2 + \frac{z^4}{16} + \frac{z^5}{45} - \frac{z^6}{720} - \frac{z^8}{10080} - \frac{z^{10}}{151200},$$

$$y_1(z) = +\frac{-73}{1152}z^4 - \frac{323}{14400}z^5 + \frac{z^6}{720} - \frac{z^8}{10080} - \frac{z^9}{136080} - \frac{z^{10}}{907200} + \frac{29}{59875200}z^{11} + \frac{23}{119750400}z^{12} + \frac{37}{778377600}z^{13} + \frac{41}{58118860800}z^{14} + \frac{1}{851350500}z^{15} + \frac{197}{10461394944000}z^{16} - \frac{1}{144353664000}z^{17} - \frac{43}{24251415552000}z^{18} - \frac{1}{4430547072000}z^{19} - \frac{1}{50634823680000}z^{20} - \frac{1}{488663280000}z^{21} - \frac{43}{81876509890560000}z^{22} + \frac{1}{73849401077760000}z^{24} + \frac{1}{3789640318464000000}z^{26},$$

and so on

$$y_{ADM} = \sum_{j=0}^{2} y_j(z) = y_0(z) + y_1(z) + y_2(z) \cong 1 + z^2, \tag{37}$$

Example (3)

The fifth order nonlinear IDE of Volterra– Fredholm is

$$.y^{(5)}(z) = -ze^{3z} + \int_0^z e^{3z-3\tau}y^3(\tau)d\tau + \int_0^z e^{z-\tau}y(\tau)d\tau, 0 \leq z \leq 1, \tag{38}$$

where

$$y(0) = y'(0) = y''(0) = y'''(0) = y^{(4)}(0) = 1\ .$$

To both sides of (38), we take the Aboodh transform and we get

$$A\left[y^{(5)}(z)\right] = A\left[-ze^{3z}\right] + A\left[\int_0^z e^{3z-3\tau}y^3(\tau)d\tau\right] + A\left[\int_0^z e^{z-\tau}y(\tau)d\tau\right]. \tag{40}$$

Applying the initial conditions together with Theorems 2-5, we obtained

$$v^5A[y(z)] = v^3 + v^2 + v + 1 + \frac{1}{v} - \frac{1}{v(v-3)^2} + \frac{1}{v(v-3)}A[y^3(\tau)] + A\left[\int_0^z e^{z-\tau}y(\tau)d\tau\right]. \tag{41}$$

Solving the (41) for $A[y(z)]$, we get

$$A[y(z)] = \frac{1}{v^2} + \frac{1}{v^3} + + \frac{1}{-} + \frac{1}{v^6} - \frac{1}{v^6(v-3)^2} + \frac{1}{v^6(v-3)}A[y^3(\tau)] + \frac{1}{v^5}A\left[\int_0^z e^{z-\tau}y(\tau)d\tau\right]. \tag{42}$$

Substituting the series assumptions for $A[y(z)]$ as given in (15) and using the recursive relation, we get

$$A[y_0(z)] = \frac{1}{v^2} + \frac{1}{v^3} + \frac{1}{v^4} + \frac{1}{v^5} + \frac{1}{v^6},$$

$$A[y_{j+1}(z)] = -\frac{1}{v^6(v-3)^2} + \frac{1}{v^6(v-3)}A\left[A_j(t)\right] + \frac{1}{v^5}A\left[\int_0^z e^{z-\tau}y(\tau)d\tau\right],\ j \geq 0, \tag{43}$$

Table 2, e approximation numericaly_{EDM} and y_{exact} for Example2.

z	y_{exact}	y_{EDM}	Absolute error
0	1	1	0
0.1	1.01	1.009999911111103	8.888889690972235e-08
0.2	1.04	1.039998544439903	1.455560097562980e-06

0.3	1.09	1.089992462310762	7.537689237890888e-06
0.4	1.16	1.159975641732821	2.435826717883671e-05
0.5	1.25	1.249939214512404	6.078548759624880e-05
0.6	1.36	1.359871181525310	1.288184746899290e-04
0.7	1.49	1.489756063481625	2.439365183752873e-04
0.8	1.64	1.639574420447577	4.255795524235051e-04
0.9	1.81	1.809302131298738	6.978687012617613e-04
1	2	1.998909272272858	1.090727727141916e-03

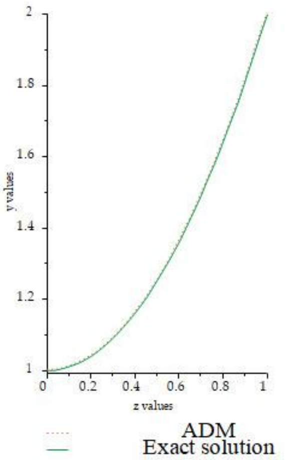

Figure 1: e Approximate Solution y_{ADM} with y_{exact} for Example 2.

where A_n' s are given by

$$\begin{aligned} A_0(\tau) &= y_0^3, \\ A_1(\tau) &= 3y_1y_0^2, \\ A_2(\tau) &= 3y_2y_0^2 + 3y_1^2y_0, \\ A_3(\tau) &= 3y_3y_0^2 + 6y_0y_1y_2 + y_1^3. \\ &\vdots \end{aligned} \tag{44}$$

We apply the inverse Aboodh transform; we have

$$y_0(z) = A^{-1}\left[\frac{1}{v^2} + \frac{1}{v^3} + \frac{1}{v^4} + \frac{1}{v^5} + \frac{1}{v^6}\right],$$

$$y_{j+1}(z) = -A^{-1}\left[\frac{1}{v^6(v-3)^2}\right] + A^{-1}\left[\frac{1}{v^6(v-3)} A\big[A_j(t)\big]\right] + A^{-1}\left[\frac{1}{v^5} A\big[\int_0^z e^{z-\tau} y(\tau)d\tau\big]\right],\ j \geq 0, \tag{45}$$

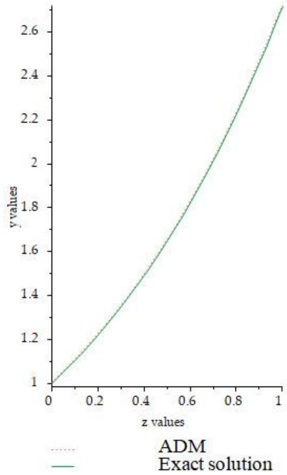

Figure 2: Comparing the numerical solution$y_{EDM}$with $y_{exact}$for Example 3.

Table *3*, e approximation numericaly_{EDM} and y_{exact} for Example*3*.

z	y_{exact}	y_{EDM}	Absolute error
00.1	11.105170918075648	11.105170918017268	05.837930139307446e – 11
0.2	1.221402758160170	1.221402756260010	1.900160251366856e – 09
0.3	1.349858807576003	1.349858792896531	1.467947230437972e – 08
0.4	1.491824697641270	1.491824634692902	6.294836829567885e – 08
0.5	1.648721270700128	1.648721075131608	1.955685198229418e – 07
0.6	1.822118800390509	1.822118304574017	4.958164925294994e – 07
0.7	2.013752707470477	2.013751613802701	1.093667775897700e− 06
0.8	2.225540928492468	2.225538745229276	2.183263192012674e – 06
0.9	2.459603111156950	2.459599057363046	4.053793903668890e – 06
1	2.718281828459046	2.718274674142005	7.154317040125591e − 06

$.y_0(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24},$

$$.y_1(z) = -\frac{1}{243} z e^{3z} + \frac{870040}{43046721} e^{3z} - \frac{3650}{729} - \frac{68}{27} z^2 - \frac{137}{162} z^3 - \frac{23}{108} z^4 - \frac{1219}{243} z - \frac{87}{8} e^{z-1} + \frac{87}{8} e^{-1} + \frac{87}{8} e^{-1} z + \frac{87}{16} e^{-1} z^2 + \frac{29}{16} e^{-1} z^3 + \frac{29}{64} e^{-1} z^4 + 5e^z - \frac{216103645}{43046721} - \frac{72555526}{14348907} z - \frac{24666787}{9565938} z^2 - \frac{4332254}{4782969} z^3 - \frac{3291049}{1275484} z^4 - \frac{114959}{4251528} z^5 - \frac{257873}{21257640} z^6 - \frac{456697}{99202320} z^7 - \frac{24853}{16533720} z^8 - \frac{338599}{793618560} z^9 - \frac{5591}{52907904} z^{10} - \frac{22123}{969978240} z^{11} - \frac{1181}{277136640} z^{12} - \frac{1633}{2401850880} z^{13} - \frac{23}{254741760} z^{14} - \frac{107}{11208637440} z^{15} - \frac{1}{1358622720} z^{16} - \frac{1}{30795448320} z^{17}. \quad (46)$$

Now, the solution for the (38) will be the following:

$$y_{EDM}(z) = \sum_{j=0}^{2} y_j(z) = y_0(z) + y_1(z) + y_2(z), \quad (47)$$

This leads to the precise answer y (z)=e^z, as seen in Figure 2.

Figure 2 demonstrates that the approximations are quite close to the actual values, and Table 3 provides an evaluation of the numerical outcomes and the precise answer.

REFERENCES

1. A M. Wazwaz, Linear And Nonlinear Integral Equations Methods And Applications, Springer, Berlin, Germany, 2011.
2. A M. Wazwaz, A First Course In Integral Equations, World Scientific, New Jersey, NY, USA, 1997.
3. B. Asady And M. T. Kajani, "Direct Method For Solving Integro Diferential Equations Using Hybrid Fourier And Block Pulse Functions," International Journal Of Computer Mathematics, Vol. 82, No. 7, Pp. 889–895, 2005.
4. H. Danfu And S. Xufeng, "Numerical Solution Of Integro- Diferential Equations By Using CAS Wavelet Operational Matrix Of Integration," Applied Mathematics And Computation, Vol. 194, No. 2, Pp. 460–466, 2007.
5. A Golbabai And M. Javidi, "Application Of He's Homotopy Perturbation Method For Nth-Order Integro-Diferential Equations," Applied Mathematics And Computation, Vol. 190, 2007.
6. A Karamete And M. Sezer, "A Taylor Collocation Method For The Solution Of Linear Integro-Differential Equations," Inter- National Journal Of Computer Mathematics, Vol. 79, No. 9, Pp. 87–1000, 2002.
7. K. Maleknejad And F. Mirzaee, "Numerical Solution Of Integro-Diferential Equations By Using Rationalized Haarfunctions Method," Kybernetes, Vol. 35, No. 10, Pp. 1735–1744,2006.
8. M. T. Kajani, M. Ghasemi, And E. Babolian, "Numerical Solution Of Linear Integro-Diferential Equation By Using Sine- Cosine Wavelets," Applied Mathematics And Computation, Vol. 180, No. 2, Pp. 569–574, 2006.
9. W. Wang, "An Algorithm For Solving The High-Order Nonlinear Volterra-Fredholm Integro-Differential Equation With Mech- Anization," Applied Mathematics And Computation, Vol. 172, No. 1, Pp. 1–23, 2006.
10. S.-Q. Wang And J.-H. He, "Variational Iteration Method For Solving Integro-Differential Equations," Physics Letters A, Vol. 367, No. 3, Pp. 188–191, 2007.
11. S. Yalcinbas And M. Sezer, ",E Approximate Solution Of High- Order Linear Volterra–Fredholm Integro Differential Equations In Terms Of Taylor Polynomials," Applied Mathematics And Computation, Vol. 112, Pp. 291–308, 2000.
12. S. H. Mohammad And E. S. Al-Rawi, "Solving Fractional Coupled Ew And Coupled Mew Equations Using Bernstein Collocation Method," Journal Of Physics: Conference Series, Vol. 1804, No. 1, Article Id 012021, 2021.
13. J. Zhao And R. M. Corless, "Compact finite Diference Method For Integro-Diferential Equations," Applied Mathematics And Computation, Vol. 177, No. 1, Pp. 271–288, 2006.
14. Journal Of Advanced Research In Computer Science And Software Engineering, Vol. 3, No. 5, Pp. 38–41, 2015.
15. A M. Wazwaz, "Anew Algorithm For Calculating Adomian Polynomials For Nonlinear Operators," Applied Mathematics And Computation, Vol. 111, Pp. 53–69, 2000.